Growth and Income Inequality: Tradeoffs and Policy Responses

Cecilia García Peñalosa
GREQAM and CNRS

Stephen J. Turnovsky
University of Washington

Very preliminary

August 2004

.

1. Introduction

The relationship between growth and income inequality has occupied the attention of the profession for some 50 years, since the appearance of Kuznets (1955) pioneering work, and is both important and controversial. Its controversy derives from the fact that it has been difficult to reconcile the different theories, especially since the empirical evidence has been inconclusive.

As detailed by Lundberg and Squire (2003), the empirical literature has tended to evolve in one of two directions. One line of research follows in the tradition of Kuznets and focuses on the question of whether more inequality enhances or inhibits the growth rate. This is addressed by running regressions of growth rates on measures of inequality and examining the signs of the relevant regression coefficients. Tests of this hypothesis by Anand and Kanbur (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), and others obtain a negative relationship. The various explanations for this include the political economy consequences of inequality (Alesina and Rodrik), the fact that inequality may harm education (Galor and Zeira, 1993), and the unequal distribution of natural resources (Gylfason and Zoega, 2003). Other studies find a positive, or at least more ambiguous, relationship; see e.g. Li and Zou (1998), Forbes (2000), and Barro (2000). In particular, Barro finds a negative relationship between inequality and growth for poorer countries, but a positive relationship in the case of richer countries.

The second approach involves examining the determinants of growth and inequality as essentially independent processes. Lundberg and Squire (2003) introduce a number of factors that may potentially influence both inequality and growth and test for their joint significance. They identify two factors that are likely to impact both variables in the same direction, thereby implying a positive relationship between income inequality and growth.[1]

But despite the controversy, one thing is clear. An economy's growth rate and its income distribution are both endogenous outcomes of the economic system. They are therefore subject to common influences, both with respect to structural changes as well as macroeconomic policies. Both structural changes, as well as policies designed to foster economic growth will almost certainly

[1] The two factors they identify are measures of civil liberty and openness.

affect agents differentially, thereby influencing the distribution of income. Likewise, policies aimed at achieving distributional objectives are likely to impact the aggregate economy's productive performance. Being between endogenous variables, the income inequality-growth relationship – whether positive or negative – will reflect the underlying common forces to which they are both reacting, and this can be understood only within the context of a consistently specified general equilibrium growth model.

The purpose of this paper is to develop a small canonical growth model in which the growth rate and income inequality are jointly determined. The framework we adopt is an extension of Romer's (1986) endogenous growth model with endogenous labor supply, with the heterogeneity of the agents, which is the source of the income inequality, stemming from their initial distribution of capital endowments. The key mechanism generating the endogenous distribution of income is the positive equilibrium relationship we derive between agents' relative wealth (capital) and their relative allocation of time between work and leisure. This relationship has a very simple intuition. Wealthier agents have a lower marginal utility of wealth. They therefore choose to work less and to enjoy more leisure, and given their relative capital endowments, this translates to an endogenously determined distribution of income.

Indeed, the role played by labor supply in this model is analogous to its role in other models of capital accumulation and growth, where it provides the crucial mechanism whereby demand shocks influence the rate of capital accumulation. For example, in the standard Ramsey model, government consumption expenditure will generate capital accumulation if only if labor is supplied elastically. With elastic labor supply it will simply crowd out an equivalent amount of private consumption. The key factor is the wealth effect and the impact this has on the labor-leisure choice. This mechanism is also central to empirical models of labor supply based on intertemporal optimization; see e.g. MaCurdy (1981).

An attractive feature of the Romer (1986) model for our purposes is that this crucial labor supply function turns out to be linear. This makes the aggregation from individuals to the aggregate – critical to the study of income distribution within a consistently derived macro equilibrium – surprisingly tractable. Indeed, we are able to represent the macroeconomic equilibrium in terms of a simple recursive structure. First the equilibrium mean growth rate and labor supply are jointly determined to ensure that rates of return are in equilibrium and that the product market clears. Second, given the initial distribution of capital among agents, the equilibrium labor supply determines the degree of income inequality.

Using this framework, we analyze the joint determination of the growth rate and income inequality and consider how this responds to various structural changes pertaining to such things as the rate of time preference, productivity, and risk. On balance, our results tend to support the view that growth-enhancing structural changes are likely to be associated with higher income equality, consistent with the recent empirical findings of Li and Zou (1998), Forbes (2000) and Lundberg and Squire (2003). But we cannot rule out the possibility of a negative relationship, especially when in reality several structural changes are occurring simultaneously. .

An equally important use of this model is for policy purposes, and to provide a framework that highlights the tradeoffs to the policy maker insofar as growth and distributional objectives are concerned. We focus on tax policy and in this regard it is important to distinguish between pre-tax (gross) and post-tax (net) income inequality, which may or may not respond in the same direction, pending the direct redistributive effects of the tax policies.

As is well-known, the externality generated by the capital stock in the Romer model implies that the competitive growth rate is too low, allowing for the possibility of welfare-improving government intervention. The first-best allocation can then be attained through suitable taxes and subsidies. With heterogeneous agents, the use of growth-enhancing policies raises the question of the impact these first-best policies may have on the distribution of income. The following general conclusion emerges from our analysis. While the increase in the growth rate and welfare associated with the first-best policies lead to an increase in pre-tax income inequality, they are also associated with lower post-tax income inequality. While the induced changes in factor prices increase pre-tax inequality, this is more than offset by the direct redistributive effect of the optimal tax structure, leading to a more equal post-tax distribution. Thus first-best fiscal policy has conflicting effects on the distributions of gross and net income, underscoring the need to distinguish between them.

The attainment of the first-best optimum in the Romer model may involve dramatic changes in income distribution and these are likely to be politically infeasible. Thus as a second application of tax policy we compare the growth and distributional consequences of financing a fixed (arbitrary) subsidy to investment by either a tax on capital income, on labor income, or on consumption. Our results highlight the sharply contrasting effects of these three different modes of finance. The relative rankings between the three forms of financing in terms of their impact on post-tax income inequality are precisely reversed from their relative impacts on pre-tax income inequality. Moreover, the relative rankings in terms of growth are markedly different from their rankings in terms of either measure of income distribution. No policy dominates in all dimensions and therefore the policy maker needs to weigh up carefully the various tradeoffs in evaluating the consequences for growth and distribution. Overall, the analysis provides support for the consumption tax, although an even more attractive policy consists of using a consumption tax and an equal-in-magnitude wage subsidy to finance the investment subsidy. Because it does not distort the labor-leisure choice, and because of the strong redistributive effect of the wage subsidy, this policy generates the greatest gains in terms of growth as well as the greatest reduction in post-tax inequality.

We should also emphasize at the outset that by adapting the Romer model, we are ignoring other important elements central to the growth-income inequality relationship, most notably human capital and education. This aspect is emphasized by Galor and Zeira (1993), Benabou (1996), and Viaene and Zilcha (2003), among others. By identifying agents' heterogeneity with their initial physical capital endowments, we are embedding distributional issues within a more traditional growth-theoretic framework. Indeed, the role of the return to capital, which is essential in that literature has largely been ignored in the recent discussions of income inequality. The argument that the return to capital is essential to understanding distributional differences has, however, been emphasized by Atkinson (2003), and is supported by recent empirical evidence for the OECD (see Checchi and García Peñalosa (2004).

There is a substantial recent literature investigating the relationship between income distribution and growth. Of this literature, our paper is related to Alesina and Rodrik (1994), Persson and Tabellini (1994), and Bertola (1993), who develop AK growth models in which agents differ in their initial stocks of capital.[2] The first two papers have, however, a very different focus as they take initial inequality as given and argue that it has a negative impact on the rate of growth. In contrast to their results, this paper emphasizes that growth and distribution are jointly determined, and presents a possible mechanism that generates a positive relationship between these two variables in line with the evidence presented by Forbes (2000). Bertola (1993) is closer to our approach in that he emphasizes how technological parameters, specifically the productivity of capital, jointly determine distribution and growth. He also examines how policies directed at increasing the growth rate affect the distribution of consumption, although his assumption of a constant labor supply implies that the distribution of income is independent of policy choices. Our approach shares with these three papers an important limitation, namely, that the assumption that agents differ only in their initial stocks of capital coupled with an AK technology implies that there are no income dynamics.[3] Finally, this paper shares a common feature with Viane and Zilcha (2003) in the sense that both relate the growth-inequality relationship to underlying structural characteristics, in a general equilibrium context, albeit a somewhat different one.

The paper is organized as follows. Sections 2 and 3 present the structure of the model and derive the macroeconomic equilibrium. Section 4 examines the determinants of the distribution of income and Section 5 analyzes the relationship between growth and inequality in response to specified structural changes. Section 6 employs the framework to address issues in taxation. It starts by obtaining the first-best optimum, and shows that the competitive growth rate is too low. It is followed by an analysis of first-best taxation, and a number of second-best policies. Section 7 supplements our theoretical analysis with some numerical simulations, used to illustrate some of the distributional implications of the various policies. Section 8 concludes, while technical details are provided in the Appendix.

[2] See, among others, Stiglitz (1969), Bourguignon (1981), Aghion and Bolton (1997) and Galor and Tsiddon (1997), as well as the overview in Aghion, Caroli, and García-Peñalosa (1999).

[3] A more general study of heterogeneity and the dynamics of distribution in growth models can be found in Caselli and Ventura (2000).

2. The Model

2.1 Description of the decentralized economy

Technology and factor payments

Firms shall be indexed by *j*. We assume that the representative firm produces output in accordance with the Cobb-Douglas production function

$$Y_j = A(L_j K)^{\alpha} K_j^{1-\alpha} \qquad 0 < \alpha < 1 \tag{1a}$$

where K_j denotes the individual firm's capital stock, L_j denotes the individual firm's employment of labor, K is the average stock of capital in the economy, so that $L_j K$ measures the efficiency units of labor employed by the firm. The production function exhibits constant returns to scale in the private factors -- labor and the private capital stock.

All firms face identical production conditions. Hence they will all choose the same level of employment and capital stock. That is, $K_j = K$ and $L_j = L$ for all *j*, where L is the average economy-wide level of employment. The economy-wide capital stock yields an externality such that in equilibrium the aggregate (average) production function is linear in the aggregate capital stock, as in Romer (1986), namely

$$Y = AL^{\alpha} K \equiv \Omega(L) K \tag{1b}$$

where $\Omega(L) \equiv AL^{\alpha}$ and $\partial\Omega / \partial L > 0$.

We assume that the wage rate and the return to capital, *r*, are determined by their respective marginal physical products. Differentiating the production function and given that firms are identical, we obtain

$$\left(\frac{\partial F}{\partial L_j}\right)_{K_j=K, L_j=L} = \alpha\Omega L^{-1} K = \alpha A L^{\alpha-1} K \equiv wK \tag{2a}$$

$$\left(\frac{\partial F}{\partial K_j}\right)_{K=K, L=L} = (1-\alpha)\Omega = (1-\alpha) A L^{\alpha} \equiv r \tag{2b}$$

implying that the equilibrium return to capital is independent of the stock of capital while the wage rate is proportional to the average stock of capital, and therefore grows with the economy.[4] In addition, we have $\partial r/\partial L > 0$ and $\partial w/\partial L < 0$, reflecting the fact that more employment raises the productivity of capital but lowers that of labor.

Consumers

There is a mass 1 of infinitely-lived agents in the economy. Consumers are indexed by *i* and are identical in all respects except for their initial endowment of capital, K_{i0}. Since the economy is growing, we are interested in the share of individual *i* in the total stock of capital, k_i, defined as $k_i \equiv K_i/K$. Relative capital has a distribution function $G(k_i)$, mean $\sum_i k_i = 1$, and variance σ_k^2.

All agents are endowed with a unit of time that can be allocated either to leisure, l_i or to work, $1 - l_i \equiv L_i$. A typical consumer maximizes expected lifetime utility, assumed to be a function of both consumption and the amount of leisure time, in accordance with the isoelastic utility function

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t) l_i^\eta\right)^\gamma e^{-\beta t} dt, \quad \text{with } -\infty < \gamma < 1, \eta > 0, \gamma\eta < 1 \tag{3}$$

where $e \equiv 1/(1-\gamma)$ equals the intertemporal elasticity of substitution. The preponderance of empirical evidence suggests that this is relatively small, certainly well below unity, so that we shall assume $\gamma < 0$. The parameter η represents the elasticity of leisure in utility. This maximization is subject to the agent's capital accumulation constraint

$$(1-s)\dot{K}_i = (1-\tau_k) r K_i + (1-\tau_w)(1-l_i) w K - (1+\tau_c) C_i \tag{4}$$

where *s* denotes a subsidy to investment in physical capital, τ_k, τ_w, and τ_c denote the tax rates on capital income, labor income, and consumption, respectively. With the equilibrium wage rate being tied to the aggregate capital stock, we observe from (4) that the individual's rate of capital accumulation depends on the aggregate stock of capital, which the individual takes as given.

Government policy

---

[4] Intuitively, in a growing economy, with the labor supply fixed, the higher income earned by labor is reflected in higher returns, whereas with capital growing at the same rate as output, returns to capital remain constant.

The government balances the public budget each instant tin accordance with the constraint

$$s\dot{K} = \tau_c C + \tau_k rK + \tau_w (1-l)wK \tag{5}$$

where C denotes aggregate consumption, l is the economy-wide average leisure time, so that $(1-l)wK$ denotes the aggregate wage bill. Note that some of the tax rates may be negative, in which case they become subsidies, along with the investment subsidy.

2.2 Consumer optimization

The consumer's formal optimization problem is to choose her rate of consumption, leisure, and rate of capital accumulation to maximize (3) subject to the accumulation equation (4). The corresponding first-order conditions are

$$C_i^{\gamma-1} l_i^{\eta\gamma} = \frac{1+\tau_c}{1-s}\lambda_i \tag{6a}$$

$$\frac{\eta}{l} C_i^{\gamma} l_i^{\eta\gamma-1} = \frac{1-\tau_w}{1-s} wK\lambda_i \tag{6b}$$

$$r\left(\frac{1-\tau_k}{1-s}\right) = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

where λ_i is agent *i*'s shadow value of capital, together with the transverality condition

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{6d}$$

These optimality conditions are standard. In equations (1) and (2) we have defined *w*, *r*, and Ω. There we have expressed them as functions of equilibrium employment, L, but assuming that the aggregate labor market clears, yields

$$\sum_j L_j = L = 1 - l = \sum_i (1-l_i) \tag{7}$$

and we can equally well write them as functions of $(1-l)$, namely

$$w=\alpha A(1-l)^{\alpha-1}=\Omega(1-l)^{-1},\ r=(1-\alpha)A(1-l)^{\alpha}=(1-\alpha)\Omega \tag{8}$$

implying

$$w(1-l)+r=A(1-l)^{\alpha}=\Omega .$$

2.3 Derivation of macroeconomic equilibrium

From the optimality conditions, together with the individual's accumulation equation, and the corresponding conditions for the aggregate economy, we shall derive the macroeconomic equilibrium, showing that the economy is in fact always on its balanced growth path. We begin by dividing (6b) by (6a) to obtain

$$\eta\frac{C_i}{K_i}=\left(\frac{1-\tau_w}{1+\tau_c}\right)w(l)l_i\frac{K}{K_i} \tag{9}$$

while we may write the individual's accumulation equation (4) in the form

$$\begin{aligned}\psi_i\equiv\frac{\dot{K}_i}{K_i}&=r(l)\left(\frac{1-\tau_k}{1-s}\right)+\left(\frac{1-\tau_w}{1-s}\right)(1-l_i)w(l)\frac{K}{K_i}-\left(\frac{1+\tau_c}{1-s}\right)\frac{C_i}{K_i}\\&=r(l)\left(\frac{1-\tau_k}{1-s}\right)+\left(\frac{1-\tau_w}{1-s}\right)w(l)\frac{K}{K_i}\left((1-l_i)-\frac{l_i}{\eta}\right)\end{aligned} \tag{10}$$

Taking the time derivative of (6a) and combining with (6c) implies

$$(\gamma-1)\frac{\dot{C}_i}{C_i}+\eta\gamma\frac{\dot{l}_i}{l_i}=\frac{\dot{\lambda}_i}{\lambda_i}=\beta-r(l)\left(\frac{1-\tau_k}{1-s}\right)\qquad\text{for each } i \tag{11}$$

The important point about (11) is that each agent, irrespective of her capital endowment, chooses the same growth rate for her shadow value of capital. Taking the time derivative of (9) implies

$$\frac{\dot{C}_i}{C_i}-\frac{\dot{l}_i}{l_i}=\frac{w'(l)l}{w(l)}\frac{\dot{l}}{l}+\frac{\dot{K}}{K} \tag{12}$$

Now consider equations (11) and (12) for individuals *i* and *k*. We obtain

$$(\gamma-1)\left(\frac{\dot{C}_i}{C_i}-\frac{\dot{C}_k}{C_k}\right)+\eta\gamma\left(\frac{\dot{l}_i}{l_i}-\frac{\dot{l}_k}{l_k}\right)=0$$

$$\left(\frac{\dot{C}_i}{C_i}-\frac{\dot{C}_k}{C_k}\right)-\left(\frac{\dot{l}_i}{l_i}-\frac{\dot{l}_k}{l_k}\right)=0$$

from which we infer

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}_k}{C_k};\ \frac{\dot{l}_i}{l_i}=\frac{\dot{l}_k}{l_k} \qquad \text{for all } i, k \tag{13}$$

That is, all agents will choose the same growth rate for consumption and leisure.

Now turn to the aggregates. Summing (9) over all agents and noting that $\sum_i k_i = 1,\ \sum_i l_i = l$, the aggregate economy-wide consumption-capital ratio is

$$\eta\frac{C}{K}=\left(\frac{1-\tau_w}{1+\tau_c}\right)w(l)l \tag{9'}$$

while summing over (10) yields the aggregate accumulation equation

$$\psi\equiv\frac{\dot{K}}{K}=r(l)\left(\frac{1-\tau_k}{1-s}\right)+\left(\frac{1-\tau_w}{1-s}\right)(1-l)w(l)-\left(\frac{1+\tau_c}{1-s}\right)\frac{C}{K}$$

$$=r(l)\left(\frac{1-\tau_k}{1-s}\right)+\left(\frac{1-\tau_w}{1-s}\right)w(l)\left((1-l)-\frac{l}{\eta}\right) \tag{10'}$$

In addition, (13) implies that average consumption, C, and leisure, l, also grow at their respective common growth rates, namely

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}}{C};\ \frac{\dot{l}_i}{l_i}=\frac{\dot{l}}{l} \qquad \text{for all } i \tag{13'}$$

The remainder of our derivation is to show that in equilibrium $l(t),\ l_i(t),\ k_i(t)\equiv K_i(t)/K(t)$ are constant through time, so that the economy is in fact always on its balanced growth path, just as it is with identical agents; see Turnovsky (2000). To show this substitute (10'), (12), and (13') into (11) expressing it by the following differential equation in l:

$$\frac{dl(t)}{dt}=\frac{G(l)}{F(l)} \tag{14}$$

where $$F(l) \equiv [1-\gamma(1+\eta)]\frac{1}{l}+\frac{(1-\gamma)(1-\alpha)}{1-l}>0$$

$$G(l) \equiv \left[\left(\frac{1-\tau_k}{1-s}\right)(1-\alpha)-(1-\gamma)\left(\left(\frac{1-\tau_k}{1-s}\right)(1-\alpha)+\left(\frac{1-\tau_w}{1-s}\right)\alpha\left(1-\frac{l}{(1-l)\eta}\right)\right)\frac{Y}{K}-\beta\right]$$

Because time is bounded, in steady-state equilibrium $\dot{l}=0$, with the corresponding stationary level of l being determined where $G(\bar{l})=0$. Thus the linearized dynamics of l about that point are represented by

$$\frac{dl(t)}{dt}=\frac{G'(\bar{l})}{F(\bar{l})}(l(t)-\bar{l}) \tag{15}$$

It is straightforward to establish that $G'(\bar{l})>0$ under any plausible conditions, in which case (14) is an unstable differential equation.[5] The only solution consistent with the eventual attainment of steady state is for $l(t)$ to be constant at all points of time.[6] It then follows from (13') that $l_i(t)$ is also constant over time.

The next step is to combine (10') and (10) to yield the following differential equation in the relative capital stock, $k_i(t) \equiv K_i(t)/K(t)$, namely

$$\dot{k}_i(t)=w(l)\left(\frac{1-\tau_w}{1-s}\right)\left[\left(1-l_i-\frac{l_i}{\eta}\right)-\left(1-l-\frac{l}{\eta}\right)k_i(t)\right] \tag{16}$$

This equation describes the potential evolution of the relative wealth (capital), starting from the initial endowment k_0. With l_i, l both constants this is a simple linear equation, the properties of which will depend the coefficient of $k_i(t)$, which we can determine from the transversality condition. If (6d) holds for all individuals it implies the aggregate condition

$$\lim_{t\to\infty}\lambda K e^{-\beta t}=0 \tag{6d'}$$

---

[5] For example, assuming a uniform income tax rate, $\tau_k=\tau_w$, a sufficient condition to ensure $G'>0$ is for $e<\frac{1+1/\eta}{1-\alpha}$. Taking $\alpha=0.6, \eta=1.75$ as plausible representations of the empirical evidence, this would permit the intertemporal elasticity of substitution to be as high as 3.9, well beyond any existing estimates.

[6] For more general production structures, particular involving two-sector economies it is possible that (15) is stable, giving rise to potential problems of indeterminate equilibria.

With l constant, (10') and (11) imply that λ and K both grow at constant rates. It is then straightforward to show that (6d') will be met if and only if

$$r\left(\frac{1-\tau_k}{1-s}\right)>\psi \tag{17}$$

i.e. the equilibrium rate of return on capital must exceed the equilibrium growth rate. It then follows from the two equations in (10') that the transversality condition can be further written in the following two equivalent ways[7]

$$(1+\tau_c)\frac{C}{K}>(1-\tau_w)w(1-l) \tag{18a}$$

$$l>\frac{\eta}{1+\eta}. \tag{18b}$$

The first equation asserts that part of income is consumed, while the latter imposes the restriction on leisure that ensures that this will be the case.

Now returning to (16) we see from (18b) that the coefficient of k_i is positive implying that the only solution consistent with long-run stability and the transversality condition is that the right hand side of (16) be zero, so that $\dot{k}_i=0$ for all time. Since k_i reflects capital stocks that evolve gradually over time, this is accomplished by agents selecting their respective leisure, l_i, in accordance with the "relative labor supply" function

$$l_i-l=\left(l-\frac{\eta}{1+\eta}\right)\left(k_i-1\right) \tag{19}$$

Thus the transversality condition (18b) yields a positive relationship between relative wealth and leisure. Setting l_i in accordance with (19) implies $\dot{k}_i\equiv 0$, so that the relative wealth position of agents, k_i, is unchanging over time. The capital stock of all agents grows at the same rate, so that at any point in time, the share of agent *i*, k_i, remains equal to her initial share $k_{i,0}$, say. Moreover, it follows from (11), (12), and (13) that individual and aggregate consumption also grow at the same common rate:

[7] In the absence of labor income, this latter condition reduces to the well known condition $C/K>0$.

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} = \frac{\dot{K}_i}{K_i} = \frac{\dot{K}}{K} \equiv \psi = \frac{r(l)\left(\frac{1-\tau_k}{1-s}\right) - \beta}{1-\gamma} \tag{20}$$

The relationship 19) is the crucial mechanism whereby the agent's initial relative endowment of capital impacts on the distribution of income. Wealthier agents have a lower marginal utility of wealth. They therefore choose to supply less labor and to "buy" more leisure. In effect, they compensate for their larger capital endowment, and the higher growth rate it would support, by providing less labor, thereby having an exactly offsetting effect on the growth rate.

The role that the elasticity of labor supply is playing in the determination of income distribution is analogous to that it plays in other similar growth models. For example, government consumption expenditure will have capital accumulation effects in the Ramsey model, and growth effects in the Romer model if and only if labor is supplied elastically. In both cases it is wealth effects that are driving the underlying responses.

3. Macroeconomic equilibrium

The above argument implies that the economy is always on its balanced growth path, the key equilibrium relationships of which can be summarized by

Equilibrium growth rate

$$\psi = \frac{r\left(\frac{1-\tau_k}{1-s}\right) - \beta}{1-\gamma} \tag{21a}$$

Individual consumption-capital ratio

$$\frac{C_i}{K_i} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l_i}{k_i} \tag{21b}$$

Aggregate consumption-capital ratio

$$\frac{C}{K} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c} l \tag{21c}$$

Individual Budget Constraint

$$\psi = r\frac{1-\tau_k}{1-s} + w\frac{1-\tau_w}{1-s}\frac{1-l_i}{k_i} - \frac{1+\tau_c}{1-s}\frac{C_i}{K_i} \tag{21d}$$

Goods market equilibrium

$$\psi = \Omega - \frac{C}{K} \tag{21e}$$

Government budget constraint

$$\tau_k r + \tau_w w(1-l) + \tau_c \frac{C}{K} = s\psi \tag{21f}$$

Recalling the definitions of $r(l)$, $w(l)$, and $\Omega(l)$, and given k_i, these equations jointly determine the individual and aggregate consumption-capital ratios, C_i/K_i, C/K, the individual and aggregate leisure times, l_i, l, average (common) growth rate, ψ, and one of the fiscal instruments given the other three policy parameters.

Using (21a), (21c), and (21e), the macroeconomic equilibrium of the economy can be summarized by the following pair of equations that jointly determine the equilibrium mean growth rate, ψ, and average leisure time, l:

RR $$\psi = \frac{(1-\alpha)\Omega(l)(1-\tau_k)/(1-s) - \beta}{1-\gamma}, \tag{22a}$$

PP $$\psi = \Omega(l)\left[1 - \frac{\alpha}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l}{1-l}\right]. \tag{22b}$$

The first equation describes the relationship between ψ and l that ensures the equality between the risk-adjusted rate of return to capital and return to consumption. The second describes the combinations of the mean growth and leisure that ensure product market equilibrium holds. We shall focus our attention on solutions that are not only viable, in the sense of satisfying the transversality condition, but also generate positive equilibrium growth. From (22b) and (18b'), the equilibrium solution for l must lie within the range:

$$\frac{\eta(1+\tau_c)}{\alpha(1-\tau_w)+\eta(1+\tau_c)} > l > \frac{\eta}{1+\eta} \tag{23}$$

3.1 The laissez-faire economy

Setting the tax rates and the subsidy to zero, the equilibrium mean growth rate and leisure in the laissez-faire economy are determined by the following pair of equations:

RR: $$\psi = \frac{(1-\alpha)\Omega(l)-\beta}{1-\gamma}, \tag{22a'}$$

PP: $$\psi = \Omega(l)\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right), \tag{22b'}$$

These RR and PP locuses are depicted in Figure 1, and their formal properties are discussed in the Appendix. First, note that equation PP is always decreasing in *l*, reflecting the fact that more leisure time reduces output, thus increasing the consumption-output ratio and having an adverse effect on the growth rate of capital. In addition, the RR curve is also decreasing in *l*. Intuitively, a higher fraction of time devoted to leisure reduces the productivity of capital, requiring a fall in the return to consumption. This is obtained if the growth of the marginal utility of consumption rises, that is, if the balanced growth rate falls. Both schedules are concave, and sufficient conditions for a unique equilibrium exists to exist are

$$\alpha-\gamma+\frac{\beta}{A}>0\,; \quad 1+\frac{1-\alpha l}{\eta(1-l)}>\frac{1-\alpha}{1-\gamma} \tag{24}$$

which are certainly met if $\gamma \le 0$, and hold under much weaker conditions as well.

4. The Distribution of Income and Welfare

We now consider the relative income of an individual having capital stock K_i. Her gross income is simply $Y_i = rK_i + wK(1-l_i)$, while average economy-wide income is $Y = rK + wK(1-l)$. Using equation (19) to substitute for the individual's labor supply, we can write the relative income of individual *i*, $y_i \equiv Y_i/Y$, as

$$y_i(l,k_i) = k_i + \frac{w}{(1+\eta)\Omega}(1-k_i) = k_i + \frac{\alpha}{(1+\eta)(1-l)}(1-k_i) \tag{25}$$

which we may express more compactly as:

$$y_i(l,k_i) = 1-\rho(l)(1-k_i), \quad \text{where} \quad \rho(l) \equiv 1 - \frac{\alpha}{(1+\eta)(1-l)}, \tag{25'}$$

Equation (25') emphasizes that the distribution of income depends upon *two* factors, the initial (unchanging) distribution of capital, and the equilibrium allocation of time between labor and leisure, insofar as this determines factor rewards. The net effect of an increase in initial wealth on the relative income of agent *i* is given by $\rho(l)$. As long as the laissez-faire equilibrium is one of positive growth, it is straightforward to show that[8]

$$0 < \rho(l) < 1 \tag{26}$$

Thus relative income is strictly increasing in k_i, indicating that although richer individuals choose a lower supply of labor, this effect is insufficiently strong to offset the impact of their higher capital income. Consequently, the standard deviation of income across the agents, σ_y, which provides a convenient measure of income inequality, is less than their (unchanging) variability of capital, σ_k.

The second point to note is that we can rank different outcomes according to inequality without needing any information about the underlying distribution of capital. For a given distribution of capital, structural or policy changes affect the distribution of income solely through their impact on relative prices, as captured by $\rho(l)$. Correia (1999) has shown that when agents differ only in their endowment of one good, there exists an ordering of outcomes by income inequality, as measured by second-order stochastic dominance.[9] That ordering is determined by equilibrium prices, and is independent of the distribution of endowments.

The DD locus in the lower panel of Fig. 1 illustrates the relationship between the standard

---

[8] The fact that $\rho(l) < 1$ is immediate from its definition in (25'). The case $\rho(l) < 0$ implies that income is negatively related to capital (wealth) and can be ruled out as perverse. In any event, it is straightforward to show that if the equilibrium is one of positive growth, (22b) suffices to ensure that $\rho(l) > 0$. We should also note that the policy maker could set tax rates so drive $\rho(l)$ to zero, if he wishes to ensure that all agents have the same pre-tax income. But this will require a negative equilibrium growth rateto offset the differential in the initial capital endowments.

[9] Her results also require that the economy be amenable to Gorman aggregation, which is the case in our setup.

deviation of relative income, σ_y, our measure of income inequality, and the standard deviation of capital endowments, σ_k, namely

$$\textbf{DD} \qquad \sigma_y = \rho(l)\sigma_k \tag{22c}$$

Given the standard deviation of capital, σ_k, the standard deviation of income is a decreasing and concave function of aggregate leisure time. This is because as leisure increases (and labor supply declines) the wage rate rises and the return to capital falls, compressing the range of income flows between the wealthy with large endowments of capital and the less well endowed. Thus, having determined the equilibrium allocation of labor from the upper panels in Fig. 1, (22c) determines the corresponding unique variability of income across agents. We may summarize this with

> **Proposition 1:** Given the initial distribution of capital across agents, fiscal policy and structural changes influence the before tax (gross) distribution of income through their effect on labor supply. However, changes in the structural parameters, α, η, have additional direct distributional effects.

Because taxes also have direct redistributive effects, we need to distinguish between the *before-tax* and *after-tax* distribution of income. Indeed, much discussion of income inequality draws the distinction between pre-tax and post-tax Gini coefficients. We therefore define the agent's after-tax (or net) relative income as

$$y_i^N(l,k_i,\tau_k,\tau_w) \equiv \frac{r(1-\tau_k)k_i + w(1-\tau_w)(1-l_i)}{r(1-\tau_k)+w(1-\tau_w)(1-l)} = 1-\rho^N(l,\tau_w,\tau_k)(1-k_i) \tag{27a}$$

where, ρ^N summarizes the distribution of after-tax income and is related to corresponding before-tax measure, $\rho(l)$, by

$$\rho^N(l,\tau_w,\tau_k) = \rho(l) + \left(1-\rho(l)\right)(1-\alpha)\frac{(\tau_w-\tau_k)}{\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)} \tag{27b}$$

with the standard deviation of after-tax income given by

$$\sigma_y^N = \rho^N(l,\tau_w,\tau_k,\alpha)\sigma_k \tag{27c}$$

From (27a) and (27b) we see that fiscal policy exerts two effects on the after-tax income distribution. First, by influencing *gross* factor returns it influences the equilibrium supply of labor, l, and therefore the before-tax distribution of income, as summarized by $\rho(l)$. In addition, it has a direct redistributive effect, which is summarized by the second term on the right hand side of (23b). The dispersion of pre-tax income across agents will exceed the post-tax dispersion if and only if $\tau_k > \tau_w$, and be smaller otherwise. In affluent OECD countries, such as US and Canada for example, ρ typically exceeds ρ^N by about 3-4 percentage points, reflecting the progressivity of the tax structure characteristic of such economies.[10]

It is also of importance to determine the relative impacts of policy and structural changes on pre-tax and post-tax income inequality. From (25) and (27) we can derive:

$$\frac{1-\rho^N}{1-\rho} = \frac{1-\tau_w}{\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)} > 0 \tag{28a}$$

$$\frac{\partial \rho^N}{\partial X} = \left(\frac{1-\rho^N}{1-\rho}\right)\frac{\partial \rho}{\partial l}\frac{\partial l}{\partial X} \tag{28b}$$

where X denotes a structural change that operates entirely through factor returns (e.g. β, A but not α). In addition, we can establish the following effects of uncompensated income tax changes (i.e financed by lump-sum taxation)[11]

$$\frac{\partial \rho^N}{\partial \tau_k} = \left(\frac{1-\rho^N}{1-\rho}\right)\frac{\partial \rho}{\partial l}\frac{\partial l}{\tau_k} - \frac{(1-\rho)(1-\alpha)(1-\tau_w)}{\left[\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)\right]} \tag{29a}$$

$$\frac{\partial \rho^N}{\partial \tau_w} = \left(\frac{1-\rho^N}{1-\rho}\right)\frac{\partial \rho}{\partial l}\frac{\partial l}{\tau_w} + \frac{(1-\rho)(1-\alpha)(1-\tau_k)}{\left[\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)\right]} \tag{29b}$$

From these expressions we can derive the following proposition.

Proposition 2: **(i)** Before-tax income inequality will be greater or less than after-tax income inequality according to whether $\tau_k \gtrless \tau_w$.

---

[10]Again, we rule out $\rho^N < 0$ as a perverse case implying a negative relationship between wealth and after tax income. It can effectively be ruled out for any plausible tax configuration.

[11] Although we have not introduced lump-sum taxation explicitly, it is easy to do, in which case changes in the two distortionary tax rates have the distributional effects noted in (29).

(ii) A change in the economic structure, other than α, has the same *qualitative* effect on after-tax income inequality, as it does on before-tax income inequality. Its *magnitude* will be greater if and only if before-tax inequality exceeds after-tax inequality.

(iii) The redistributive effect of an uncompensated increase in τ_k *reinforces* the negative effect on the before-tax income distribution. In contrast, the redistributive effect of an uncompensated increase in τ_w *offsets* the negative effect on the before-tax income distribution. Indeed, it is possible for it to dominate, so that pre-tax and post-tax incomes respond in opposite ways.

Lastly, we compute individual welfare. By definition, this equals the value of the intertemporal utility function (3) evaluated along the equilibrium growth path. Performing this calculation the optimized level of utility for an agent starting from an initial stock of capital, K_{i0}, can be expressed as

$$X(K_{i0}) = \frac{1}{\gamma}\frac{\left((C_i/K_i)l_i^{\eta}\right)^{\gamma}}{\beta-\gamma\psi}K_{i,0}^{\gamma} \tag{30}$$

The welfare of individual *i* relative to that of the individual with average wealth is then

$$x(k_i) = \frac{\left(C_i/K_i\right)^{\gamma}}{\left(C/K\right)^{\gamma}}\frac{l_i^{\eta\gamma}}{l^{\eta\gamma}}k_i^{\gamma} = \left(\frac{l_i}{l}\right)^{(1+\eta)\gamma} \tag{31}$$

where the second term has been obtained by substituting for the consumption-capital ratio. Using equations (18), we can express relative welfare as

$$x(k_i) = \left[1+\left(1-\frac{\eta}{1+\eta}\frac{1}{l}\right)\left(k_i-1\right)\right]^{\gamma(1+\eta)}. \tag{31'}$$

Consider now two individuals having relative endowments $k_2 > k_1$. Individual 2 will have a higher mean income. The transversality condition (13") implies that if $\gamma > 0$, then their relative welfare satisfies $x(k_2) > x(k_1) > 0$, while if $\gamma < 0$, $x(k_1) > x(k_2) > 0$. However, in the latter case absolute welfare, as expressed by (30) is negative. Thus in either case, the better endowed agent will have the higher absolute level of welfare, so that the distribution of welfare moves together with that of income.

With heterogeneous agents the conventional criterion is to evaluate the social benefits of policy in terms of the utilitarian welfare function, defined as the sum of the welfare of the individual agents in the economy.

$$\Lambda(K_0) \equiv X(K_0)\int_0^\infty x(k_i)G(k_i)dk_i \tag{32a}$$

where

$$X(K_0) = \frac{1}{\gamma}\frac{\left((C/K)l^\eta\right)^\gamma}{\beta-\gamma\psi}K_0^\gamma \tag{32b}$$

denotes the welfare of the average agent in the economy. In general, the evaluation of (32a) is intractable. One case that is tractable is if we assume that k_i is uniformly distributed over the interval $[2-\bar{k} < k_i < \bar{k}]$, so that

$$G(k_i) \equiv \frac{1}{2(\bar{k}-1)}$$

Simplifying further by assuming $\bar{k} = 2$, evaluating (32a) leads to

$$\Lambda(K_0) = \frac{1}{\gamma}\frac{\left((C/K)l^\eta\right)^\gamma}{\beta-\gamma\psi}K_0^\gamma \frac{\left[\left(2-\frac{\eta}{(1+\eta)}\right)^{1+\gamma(1+\eta)} - \left(\frac{\eta}{(1+\eta)}\right)^{1+\gamma(1+\eta)}\right]}{2\left[1+\gamma(1+\eta)\right]\left[1-\frac{\eta}{(1+\eta)l}\right]} \tag{33}$$

Utilitarian welfare comprises two components: the utility of the median individual, adjusted by a term that takes account of the dispersion across agents. Note that in the case of the logarithmic utility function $(\gamma = 0)$ (33) reduces to $X(K_0)$. Otherwise, if $\gamma > 0$, $0 < \Lambda(K_0) < X(K_0)$, while if $\gamma < 0$, $0 > X(K_0) > \Lambda(K_0)$. In either case, the utility of the median agent overstates the utilitarian level of utility.

5. The Relationship between Inequality and Growth

As emphasized at the outset, and as our analysis has illustrated, inequality and growth are jointly determined. We now use Fig 1 to explore the relationship in response to two structural shocks: (i) an increase in savings, generated by a reduction in the rate of time discount, β, and (ii) an increase in productivity, *A*. For simplicity we focus on the laissez-faire economy.

5.1 Decrease in rate of time preference, β

This is illustrated in Fig. 2. The intersection of the PP and RR curves at Q determines the initial equilibrium fraction of time devoted to leisure, $\bar{l}$, and the corresponding growth rate, $\bar{\psi}$. Given $\bar{l}$, the corresponding point M on the DD curves determines the corresponding degree of income inequality, $\bar{\sigma}_y$. A decrease in β rotates the RR curve upward, to R'R, with PP and DD remaining unchanged. As a result, the new equilibrium shifts to M' and Q', with a higher growth rate, less leisure, and a greater income inequality.

The intuition for this response is straightforward. Given labor supply, the decline in impatience associated with the lower rate of time preference leads to an increase in savings and an immediate increase in the growth rate. This is represented by the move from Q to S on R'R. But the higher growth rate implies higher future wages, and hence higher consumption for any extra time spent at work. It therefore reduces leisure, increasing the supply of labor, raising the return to capital and causing a further increase in the growth rate, as measured by the move SQ' along the R'R curve. The reallocation of time in turn, affects the distribution of income. The increase in labor supply raises the return to capital and decreases that to labor. Since labor is more equally distributed than capital, the income gap between any two individuals rises and income inequality increases.

5.2 Increase in productivity, *A*

From Fig. 3 we see that this leads an upward shift in both the RR and PP curves, the latter rotating about the point T, with the equilibrium shifting from Q and M, to Q' and M', respectively. The upward shift in the RR curve has the effects described above, and the same intuition applies. However, the upward shift in the PP curve (for $\psi > 0$) has offsetting effects, causing an increase in *l* accompanied by a lower growth rate, ψ. On impact, given labor supply and output, this raises the growth rate and the return on consumption, causing agents to increase consumption and leisure over work. This causes a reduction in output and the growth rate, leading to a reduction to the return to capital and consumption. On balance the former effect can be shown to dominate and the new equilibrium at Q' is associated with a higher growth rate and less leisure, and therefore greater income inequality.

The introduction of taxes preserves both these sets of relationships. In addition, after-tax inequality rises as well, by a greater or lesser amount depending upon whether $\tau_w \gtrless \tau_k$.

We may therefore summarize these results with the following:

> **Proposition 3:** A decrease in the rate of time preference, or an increase in productivity induces a *positive* relationship between growth and both pre-tax and post-tax income inequality.

Two points should be noted. First, the elasticity of labor supply is critical to these relationships. If labor were inelastically supplied at $l = 1$, say, the relative income of agent *i*, $y_i = (w + rk_i)/(w + r)$. In this case the AK technology results in a constant wage and interest rate, so that this expression would be unaffected by these structural changes. In our setup they matter because they affect the growth rate, and this, in turn, impacts labor supply and factor rewards.

Second, it is evident from Figure 1 that with both RR and PP being negatively sloped, any structural or policy change that induces a shift in only *one* of these curves always generates a negative relationship between growth and leisure. If further, the DD curve remains unchanged, this translates into a positive relationship between growth and income inequality. The decrease in β is one example of this and an increase in γ is another. And it is also true of an increase in any of the tax rates [with revenues rebated as lump-sum transfers]. Increases in τ_k, τ_y, τ_c all lead to a lower growth rate, more leisure, and less pre-tax income inequality.

In order to induce a negative relationship between growth and inequality the structural change must impinge on at least two curves. But even a productivity increase, which shifts both the PP and RR curves, induces an unambiguous positive growth-income inequality relationship, as we have seen. Moreover, numerical simulations based on plausible calibrations [see Table 3 below] suggest that changes in η that shift the PP and DD curves, and α which impacts on all three curves are likely to generate a positive relationship between growth and inequality.

We are thus led to speculate that for the present AK technology any single structural change is likely to generate a positive relationship between growth and income inequality. To generate a negative inequality would require the introduction of two shocks. One example of this may be a productivity increase coupled with an increase in the rate of time preference, or, as we shall now discuss, it may be induced as the result of some policy action.

6. Taxation

A familiar feature of the Romer (1986) model is that by ignoring the externality associated capital, the decentralized economy generates a sub-optimally low growth rate. This suggests that an investment subsidy that increases the growth rate will move the equilibrium closer to the social optimum. With heterogeneous agents, two questions arise. First, how to finance this subsidy if the government is concerned about inequality as well as about average welfare. An investment subsidy raises the return to capital and will tend to favor those with large capital holdings. If the subsidy were financed by a lump-sum tax, the system would redistribute away from those with lower incomes to those with higher incomes. Are there ways in which this reverse redistribution can be avoided? Second, we want to know whether the use of first-best policies has any implication for the relationship between growth and inequality. In this section we investigate these questions in some detail. We begin by deriving the first-best optimal rate of growth and allocation of labor.

6.1 The first-best optimum

Given the externality stemming from the aggregate capital stock, finding the first-best optimum amounts to solving the following problem:

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t) l_i^\eta\right)^\gamma e^{-\beta t} dt, \tag{34a}$$

subject to

$$\dot{K}_i = \left(\Omega K_i - C_i\right) \tag{34b}$$

the macroeconomic equilibrium to which is described by the equations

R'R' $$\tilde{\psi} = \frac{\Omega(\tilde{l}) - \beta}{1-\gamma} \tag{22a''}$$

PP $$\tilde{\psi} = \Omega(\tilde{l})\left[1 - \frac{\alpha}{\eta}\frac{\tilde{l}}{1-\tilde{l}}\right] \tag{22b}$$

DD $$\sigma_y = \rho(\tilde{l})\sigma_k \tag{22c}$$

$$\frac{\tilde{C}}{\tilde{K}} = \frac{\alpha\Omega(\tilde{l})}{\eta}\frac{\tilde{l}}{1-\tilde{l}} \tag{22d}$$

where the tilde denotes the first-best optimum. Note that the only difference from the solution to the laissez-faire economy is that the social rate of return to capital now takes into account the production externality and hence exceeds the private return.[12] The R'R' schedule lies above RR. Given that the PP schedule is steeper than RR, the upward shift of RR results in a higher growth rate, lower leisure, and therefore increases inequality, as can be seen from Figure 2.[13] That is $\tilde{l} < \bar{l}, \tilde{\psi} > \bar{\psi}, \tilde{\rho} > \bar{\rho}$, where the bar denotes the laissez-faire economy.

6.2 First-best taxation

Comparing the first-best optimum, described by R'R', and PP with the decentralized equilibrium, RR, PP we can see that following tax-subsidy structure will attain the optimal growth rate, leisure time, and consumption/capital ratio:

$$\frac{1-\tau_w}{1+\tau_c} = 1, \quad \text{i.e.} \quad \tau_w = -\tau_c \ , \tag{35a}$$

[12] The transversality condition (12) for the central planner's problem again reduces to (16) but is now automatically satisfied without the restriction (18b') being imposed.

[13] The reason why the social planner chooses less leisure is that there are in fact two externalities in the model. On the one hand, a greater individual stock of capital increases the aggregate level of technology. On the other, a higher labor supply raises the marginal product of capital and induces greater accumulation of capital, thus increasing the level of technology.

$$(1-\alpha)\frac{1-\tau_k}{1-s}=1, \quad \text{i.e. } s=\alpha+(1-\alpha)\tau_k\,, \tag{35b}$$

$$\frac{1-\tau_k}{1-\tau_w}=\frac{1}{1-\alpha}\left[1-\eta\frac{1-\tilde{l}}{\tilde{l}}\right], \tag{35c}$$

where the last equation is obtained from the government's budget constraint, (21f). The first two equations represent intuitive optimality conditions. The first states that any wage tax should be offset with an equivalent consumption tax so as not to distort the leisure-consumption choice. The second condition simply ensures that the private rate of return on investment must equal the social return, and for this to be so the subsidy to investment must exceed the externality by an amount that reflects any tax on capital income. Note from the third equation that unless consumption equals total income, (in which case there is zero growth), the replication of the first optimum requires differential taxes on wages and capital; $\tau_w \lessgtr \tau_k$ according to whether there is positive or negative growth.

Equations (35) indicate the existence of a degree of freedom in the optimal tax-subsidy structure. One instrument can be set arbitrarily and we shall take it to be *s*. In this case (35a) – (35c) imply the following first-best optimal tax rates:

$$\hat{\tau}_k=\frac{s-\alpha}{1-\alpha} \tag{36a}$$

$$\hat{\tau}_w=\frac{s-\eta(1-\tilde{l})/\tilde{l}}{1-\eta(1-\tilde{l})/\tilde{l}}=-\hat{\tau}_c \tag{36b}$$

from which, assuming positive growth, we may conclude the following relative magnitudes:

$$0<s<\alpha: \qquad \tau_w=-\tau_c<\tau_k<0<s \tag{37a}$$

$$\alpha<s<\eta\left(\frac{1-\tilde{l}}{\tilde{l}}\right): \qquad \tau_w=-\tau_c<0<\tau_k<s \tag{37b}$$

$$\eta\left(\frac{1-\tilde{l}}{\tilde{l}}\right)<s: \qquad 0<\tau_w=-\tau_c<\tau_k<s \tag{37c}$$

There are two things to note about the optimal tax structure. First, is that for a sufficiently small investment subsidy, it will call for subsidies to both wage income and capital income, all financed by the consumption tax. But as the investment subsidy increases, both forms of income should be taxed, with the revenues financing both the initial investment subsidy and now a subsidy to consumption. The second point is that with the economy being one of positive growth, $\tau_k > \tau_w$, so that the net tax on capital should exceed that on labor income [or the subsidy should be smaller in (37a)]. This may appear to counter conventional wisdom that argues that capital income should be untaxed, or generally taxed at a lower rate than labor income; see Judd (1985), Chamley (1986).

The intuition behind this relationship can be most easily understood, by considering the equilibrium government budget constraint (21f) when $s = 0$. Substituting for r, w, and C/K and invoking (35a,b), when it becomes

$$-\alpha = \tau_k(1-\alpha) = \tau_w\alpha\left(\frac{l}{\eta(1-l)} - 1\right)$$

In this case the government needs to raise net tax revenues equal to α to finance the subsidy to capital income. Given (i) that in equilibrium consumption exceeds labor income [the transversality condition (18a)], and (ii) the constraint $\tau_w = -\tau_c$, necessary not to distort the labor-consumption choice, it is clear that to raise positive tax revenues requires $\tau_c = -\tau_w > 0$, so that labor income must be subsidized as well. Moreover, since the consumption tax must finance the subsidy to both capital and labor it follows that $\tau_c > -\tau_k$, implying that $\tau_k > \tau_w$. Thus our analysis can be viewed as a further example in a growing body of literature challenging the conventional view that capital income should be taxed less than labor income.[14]

What is the impact of the first-best taxation system on distribution? Recall that the dispersion of gross income is given by (22c), where $\rho(l)$ is a decreasing function of leisure time. Since the first-best optimal tax policy increases the time allocated to labor, it will increase gross income inequality, i.e. $\tilde{\rho}(\tilde{l}) > \bar{\rho}(\bar{l})$. The corresponding dispersion of net income in the decentralized economy that mimics the centrally planned equilibrium is obtained by substituting the optimal tax rates, (36a), (36b), into (27b) to yield

[14]In a previous paper analyzing a two-sector economy having an informal sector that can escape taxes, we found that taxing capital income at least as much as, and possibly more heavily than, labor income could well be optimal; see García-Peñalosa and Turnovsky (2004). Other studies reaching similar conclusions, though employing very different approaches, include Fuest and Huber (2001), and Koskela and Schöb (2002).

$$\tilde{\rho}^N(\tilde{l};\hat{\tau}_w,\hat{\tau}_k) = \rho(\tilde{l}) + (1-\tilde{\rho})\left(\frac{\alpha - \eta\left((1-\tilde{l})/\tilde{l}\right)}{1+\alpha-\eta\left((1-\tilde{l})/\tilde{l}\right)}\right) \tag{38}$$

The striking aspect about (38) is that the distribution of net income is *independent* of the (arbitrary) choice of fiscal instruments employed to achieve this objective. As long as the equilibrium is one of positive growth, then $\tilde{\rho}^N < \tilde{\rho}$, and net income is less dispersed than is gross income. Indeed, it is possible for the direct redistributive effect of the first-best optimal tax to dominate the indirect effect due to changes in factor prices, so that the net income inequality is reduced below that of the laissez-faire economy, i.e. $\tilde{\rho}^N < \overline{\rho}$. Simulations presented below suggest that this is in fact a likely outcome.

When the first-best tax system is implemented, the effects of structural changes on growth and leisure are equivalent to those in the first-best economy, as can be easily verified from equations (22a"), (22b-d). As in the decentralized economy, a productivity increase or a reduction in the rate of time preference is associated with a greater supply of labor and hence with more pre-tax inequality. The effects of these structural changes on after-tax inequality are, however, ambiguous. Differentiating (38) with respect to *l*, we can see that there are two opposing effects. On the one hand, more leisure tends to reduce pre-tax inequality. On the other, (and as long as the subsidy rate is less than 1), a lower labor supply implies that a higher wage tax is required in order to finance any given subsidy rate (see (36b)), making the fiscal system less progressive. The latter may quite plausibly dominate, in which case the reduction in *l* may be associated with a decline in post-tax income inequality. This can be seen from the expressions

$$\frac{1-\tilde{\rho}^N}{1-\tilde{\rho}} = \frac{1}{1+\alpha-\eta\left((1-\tilde{l})/\tilde{l}\right)} > 0 \tag{39a}$$

$$\frac{\partial\tilde{\rho}^N}{\partial\tilde{l}} = \left(\frac{1-\tilde{\rho}^N}{1-\tilde{\rho}}\right)^2\left[(1+\alpha+\eta) - \frac{\eta}{\tilde{l}^2}\right]\frac{\partial\tilde{\rho}}{\partial\tilde{l}} \tag{39b}$$

It is evident from (39b) that the reduction in $\tilde{l}$ will dominate if and only $\hat{l} < \eta(1+\alpha+\eta)$, a condition that is quite plausibly met, as our simulations below confirm. We summarize our results with

Proposition 4: (i) The decentralized economy will replicate the first-best optimum if and only if tax rates are set in accordance with

$$\hat{\tau}_k = \frac{s-\alpha}{1-\alpha}, \qquad \hat{\tau}_w = \frac{s-\eta(1-\tilde{l})/\tilde{l}}{1-\eta(1-\tilde{l})/\tilde{l}} = -\hat{\tau}_c$$

(ii) Pre-tax income inequality in the optimal economy exceeds that in the laissez-faire economy, and as long as the equilibrium is one of steady growth, it exceeds post-tax income inequality in the optimal economy as well

(iii) In contrast to the decentralized economy, a change in the economic structure (such as an increase in productivity) that raises the growth rate, lowers leisure, and increases pre-tax income inequality, may quite plausibly reduce post-tax inequality.

6.3 Financing an investment subsidy

To attain the first-best equilibrium is likely to require the tax rates to assume extreme values, even for plausible parameter values; see Table 4 below. These will generate dramatic changes in the distribution of income that likely will render them politically infeasible. Indeed, our numerical analysis (see Table 3 below) implies differences between the gross and the net Gini coefficients of 14 to 20 Gini points, whereas actual differences in OECD countries range between 1.5 and 4 points. Thus, we now consider some less drastic policy responses, which nevertheless, as our simulations show, may still yield substantial welfare gains, but have contrasting effects on both the pre-tax and post-tax income inequality.

Subsidy to investment financed by a tax on capital income

Suppose that the fiscal authority decides to finance the subsidy to investment with a tax on capital income, alone. Setting $\tau_w = \tau_c = 0$ in the government budget constraint (24c) the required tax on capital income is:

$$\tau_k = \frac{s}{(1-\alpha)}\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right) \qquad (40a)$$

From equations (22a) we see that this policy shifts the RR schedule upward and leaves the PP schedule unchanged, increasing the growth rate and reducing leisure. The reduction in leisure increases the pre-tax degree of income inequality, $\rho(l)$. Recall that the net distribution of income was characterized by (27b). Then, taxing capital income ensures that $\rho^N(l,\tau_w\tau_k) < \rho(l)$. If the redistributive effect dominates, as our simulations suggest may plausibly occur, the after-tax inequality actually declines, relative to the laissez-faire distribution.

Subsidy to investment financed by a tax on wage income

Alternatively, the subsidy may be fully financed by a wage tax

$$\tau_w = \frac{s}{\alpha}\frac{\left(1-(\alpha/\eta)\left(l/(1-l)\right)\right)}{\left(1-(s/\eta)\left(l/(1-l)\right)\right)} \qquad (40b)$$

In this case, both the RR and PP schedules shift up, resulting in a higher growth rate and greater or lower leisure, depending on the relative shifts. The reason for the ambiguous effect on leisure is that the wage tax tends to reduce the supply of labor, while the higher growth rate tends to increase it.

The ambiguous response of labor complicates the impact on the inequality of income. First, the increase/decrease in leisure time will reduce/increase the dispersion of gross incomes, as seen from (25'). However, the required (positive) wage tax implies taxing the factor that is more equally distributed, and for any given distribution of gross incomes this raises the variability of net incomes (see (27b) above). If the policy reduces leisure time, it would then unambiguously increase pre-tax and post-tax income inequality. However, when leisure time increases, the two effects work in opposite directions: there will be a reduction in the variability of gross income, while net income inequality may increase or decrease as compared to the equilibrium without taxes.[15]

Subsidy to investment financed by a tax on consumption

As a third example, the subsidy may also be financed by setting the consumption tax equal to

[15] We can, however, see that when the subsidy rate matches the externality, $s=\alpha$, $\tau_w=1$ and $\rho^{NET}=1$ implying that the net income inequality is increased to that of the initial endowment of capital.

$$\tau_c = s\frac{\left(1-(\alpha/\eta)(l/(1-l))\right)}{\left((\alpha/\eta)(l/(1-l))-s\right)} \tag{40c}$$

in which case $\rho^N(l,\tau_w,\tau_k)=\rho(l)$. Again both schedules shift upwards, increasing the growth rate. In this case it can be shown that leisure declines, so that gross income inequality increases. Since the consumption tax has no direct redistributive effect, the gross and the net distributions of income are identical and hence net income inequality increases as well.

Tables 1 and 2 provide formal expressions for the effects of the investment subsidy on growth and distribution, under the different financing modes, together with their rankings along different dimensions. As noted in Table 2, these rankings can be sharpened somewhat by mildly strengthening the restrictions, as indicated, but consistent with our simulation results. The striking feature of these results is that they highlight the sharply contrasting consequences of the different modes of financing the subsidy.

Assume that the policy maker's objective is to raise growth and lower income inequality. The first point the table emphasizes is the need to contrast gross versus net income; indeed, the relative merits between the three modes of finance are precisely reversed insofar as these two objectives are concerned. Second, the relative rankings in terms of growth contrast markedly from the rankings in terms of either measure of income distribution. Third, while each mode of financing may be superior in terms of some criterion, it is dominated in others. For example, while financing by capital income tax has the most desirable effect on post-tax income distribution it is worst from a growth perspective. The consumption tax is best for stimulating growth, but it is dominated by the wage tax from the perspective of pre-tax income inequality and the capital tax for post-tax inequality. Financing by a wage income tax is best from the viewpoint of gross income inequality, but it is worst for post-tax inequality, and is dominated by the consumption tax insofar as stimulating growth is concerned. The table thus highlights the need for a policy maker wishing to stimulate investment in this way to take careful account of the tradeoffs along these various dimensions. We summarize these results with

Proposition 5: Suppose a policy maker wishes to stimulate investment through a subsidy. From a growth perspective, financing the subsidy using a consumption tax is superior to a wage tax, and in turn to a capital income tax. In terms of gross income inequality, the wage tax is superior to the consumption tax, which in turn dominates the capital income tax. However, these latter rankings are reversed in terms of their impact on net income inequality.

7. Numerical examples

To obtain further insights into the growth-income inequality relationship we provide some numerical examples. To do so we use the following, mostly conventional, parameter values:

Parameter Values

Production	$A = 0.75,\ \alpha = 0.60$
Preferences	$\beta = 0.04,\ \gamma = -2,\ \eta = 1.75$

The choice of production elasticity of labor measured in efficiency units implies that 60% of output accrues to labor. One consequence of the Romer technology, is that whereas this value is realistic in terms of the labor share of output, it implies an implausibly large externality from aggregate capital which implies extreme solutions for the first-best fiscal policy, discussed below. The choice of the scale parameter A = 0.75, is set to yield a plausible value for the equilibrium capital-output ratio.

Turning to the preference parameters, the rate of time preference of 4% is standard, while the choice of the elasticity on leisure, $\eta = 1.75$, is standard in the real business cycle literature, implying that about 72% of time is devoted to leisure, consistent with empirical evidence. Estimates of the intertemporal elasticity of substitution are more variable throughout the literature. Our choice 0.33 is well in the range of the empirical evidence, which with few exceptions lies in the range (0,1).

Our numerical measures of income distributions are reported in terms of the more standard Gini coefficients, as well as the standard deviation measures employed in our theoretical discussion. This involves choosing an initial distribution of wealth, which is less straightforward, as data on the distribution of wealth are difficult to obtain.[16] The choice we have made yields a Gini coefficient of around 33.3%. These compare with 39.1% and 31.1% for the US and Sweden in 1991 and 1992, respectively (from Deininger and Squire, 1996).

7.1 Laissez-faire economy

The first line of Table 3 reports the benchmark equilibrium in the laissez-faire economy for our base parameters. It indicates that 72.5% of time is allocated to leisure, yielding a growth rate of 3.3%, a standard deviation of income inequality of 0.206.[17] The welfare level of the median agent is higher than that of the utilitarian utility measure, which is a consequence of our assumption $\gamma < 0$, as noted previously. This is a plausible benchmark and $l = 0.725$ lies in the range [0.636, 0.745], consistent with (23).

Lines 2 and 3 report the effects of an increase in productivity from 0.75 to 1, and a reduction in the rate of time preference from 0.04 to 0.02. In both cases, leisure declines, growth increases, and income becomes more unequal, precisely as discussed in Section 4. Welfare changes reported are calculated as the percentage equivalent variations in the initial stock of capital of the average individual necessary to maintain the level of utility following the structural change. The 33% increase in productivity raises welfare by around 50%, while the increase in patience is actually mildly welfare-deteriorating. This is because the initial response is a decline in consumption, and although the increase in the growth rate raises consumption over time, this is dominated by the initial decline, so that in present value terms welfare falls slightly. Lines 4 and 5 present increases in α, η, that we did not discuss analytically. This is typical of simulations that we ran and in all cases they always yield a pronounced positive growth-inequality relationship.

7.2 First-best optimum

[16] We have assumed that distribution of wealth among the 5 quintiles is 0, 0, 1.2%, 12%, 86.8%, which are consistent with the data. For example, in the US in 1992 the bottom 40% of the population held 0.4% of total wealth, while the top 20% owed 83.8% of the total; see Wolff (1998).

[17] For simplicity we normalize $\sigma_k = 1$.

Table 4 summarizes the first-best equilibrium in the centrally planned economy, with the three panels corresponding to the first three of Table 3. It offers a number of insights that both reinforce and complement our analytical results. First, we see that the policy involves a substantial reduction in leisure time (between 10 and 12 percentage points), raising the growth rate dramatically (by a factor of 3!).[18] The quantities, $\Delta X, \Delta Y$ in the first row are the welfare changes (for the median agent and social welfare, respectively) relative to the benchmark economy (i.e. those in Table 2 Row 1). First-best taxation increases the welfare of the average individual in the economy by over 20% and overall welfare by around 25%.

The effects on income distribution are substantial. The large reduction in leisure time results in a large increase in pre-tax inequality, as measured by both the standard deviation and the Gini coefficient. However, the redistributive effect is strong enough to dominate this response and yield an overall reduction in net income inequality. The reduction in post-tax inequality relative to the economy without subsidies is large, amounting to about 6 Gini points.

Table 3 also illustrates the analytical results that the first-best equilibrium can be replicated by a variety of tax/subsidy configurations, each of which leads to precisely the same post-tax distribution of income. The sensitivity of the tax regime to changes in the subsidy rate is also borne out. Focusing on the first panel, in the absence of a subsidy to investment, the first best equilibrium can be sustained if income form capital and labor are subsidized at the rates of 150% and nearly 600%, respectively, while consumption is taxed correspondingly at nearly 600%![19]

This is hardly a politically viable tax structure. But the first-best equilibrium can also be attained if, more reasonably, investment is subsidized at around 86%, being financed by a tax on capital income of around 64%, leaving consumption and labor income untaxed. Or, if investment and consumption are subsidized at 90% and 30% respectively, with taxes on labor income and capital income of 30% and 75%, respectively.

[18] The implied percentage increase in labor supply is much larger, being of the order of 20%.

[19]The cause of this problem is the magnitude of the externality associated with the Romer model.If, e.g. α is reduced to say 0.2, the subsidy is drastically reduced, however, the allocation of time and the share of labor earned by output becomes less plausible. In the case of inelastically supplied labor, the allocation of the first-best optimum can require extreme taxation independent of the magnitude of the externality. For example, Bertola (1993) shows that the first best optimum can be obtained by taxing labor to subsidize income from capital, in accordance with $(1-\alpha)(1-\tau_k)=1$, $(1-\alpha)\tau_k+\alpha\tau_w=0$, which implies taxing labor at 100%, irrespective of the externality.

Panels 2 and 3 of Table 3 introduce structural changes in the optimal equilibrium. The main point of interest is that for both shocks while gross income inequality rises, net income inequality falls. This divergent response of pre- and post-tax inequality, impossible in the laissez-faire economy, is consistent with Proposition 4, part (iii). Our results suggest that this is in fact quite a plausible outcome. The welfare measures in these two panels are changes resulting from the structural changes and are therefore relative to the welfare level in panel 1. Thus for example, an increase in A from 0.75 to 1 raises welfare by around 47% in the optimal economy, as compared to 50% in the laissez-faire economy, although of course from a higher base level.

7.3 Financing an investment subsidy

Table 5 reports the numerical effects of financing a fixed 30% investment subsidy through a capital income, wage tax or a consumption tax, respectively. The rankings with respect to their effects on growth, leisure, pre-tax inequality, post-tax inequality confirm the theoretical rankings reported in Tables 1 and 2. Table 5 also confirms some of the more ambiguous effects of our theoretical findings. For example it confirms the theoretical possibility that the redistributive effects in both capital income tax and wage income tax financing dominate, so that post-tax inequality in the former declines, while in the latter it rises. In addition, the table provides the welfare gains for both the average individual and the overall utilitarian welfare function. These simulations suggest that, despite the ambiguities with respect to the various rankings, overall consumption-tax financing is the superior policy.

The last row of the table considers financing the subsidy through a combination of wage and consumption taxes. In particular, we set $\tau_w = -\tau_c$; that is, these two taxes are optimally set, although the subsidy is below the first-best level. The effect of this policy on the growth rate is stronger than in the previous three cases, the reason being that this policy does not distort the allocation of time between labor and leisure. Employing only a wage or a consumption tax tends to reduce the supply of labor, partially offsetting the effect of the subsidy. When both are used, this effect is absent. Since setting $\tau_w = -\tau_c$ results in faster growth than using only one tax, this policy generates larger welfare gains than any of the pure policies. The effect on distribution is quite significant. In contrast to financing the investment subsidy by either a wage or a consumption tax alone, financing through a combination of a consumption tax and wage subsidy reduces substantially post-tax inequality.

The investment subsidy in Table 5 is arbitrary. Table 6 summarizes a number of second best policies, whereby the policy maker sets the optimal subsidy for each of the three modes of finance. In the case where it is financed with a tax on capital income, it is able to attain the first-best optimum. In that case setting s = 85.6% and $\tau_k = 64.0$% improves welfare by over 20% for the average agent and generates the distribution of income associated with the first-best optimum. Alternatively, setting *s* = 57.3%, financed with 26% tax on wages, or *s* = 60.2% financed with a 25.4% consumption tax yield second-best optima. The interesting aspect about these latter two alternatives is that they are fairly moderate policies, in contrast to the first-best, summarized in Table 3. In particular, the consumption tax yields the major portion of the welfare gains obtained in the first-best case (19% out of a total increase in welfare of 20.6%), while having only a minimally adverse impact on income distribution. Indeed, to a policymaker concerned with maximizing the welfare of the average agent, with minimum distortionary effect on the distribution of income, this policy may be particularly attractive.

8. Concluding Comments

The relationship between growth and inequality remains largely unresolved, despite the intensive research devoted to it over the past 50 years. Empirical evidence is inconclusive, some authors finding these two variables to be negatively related, while others obtain a positive relationship; see Lundberg and Squire (2003). Indeed, the ambiguity of the empirical findings should not be too surprising when one considers that both variables are endogenous, so that their co-movements are likely to depend upon the underlying structural and policy changes driving them. To study this requires that they be analyzed within the context of a consistently specified growth model. This is important, not only to understand the relationship between them, but also in devising appropriate policy responses.

In this paper we have developed a small canonical model, extending the basic AK growth model with endogenous labor supply to agents having heterogenous initial endowments of capital. The key mechanism whereby this initial distribution of capital endowments influences the distribution of income is through their differential wealth effects, and their impact on labor supply. If capital endowments are more unequally distributed than labor endowments, any structural change that tends to increase the supply of labor and raise the relative return to capital, raises the return of the factor that is the source of the inequality, and the distribution of income becomes more unequal. This will tend to induce a positive relationship between inequality and growth. Indeed. our model tends to such a positive relationship in the sense that it always emerges in the case of independent structural changes. A negative relationship would require coordinated (multiple) structural shocks.

The endogeneity of the labor supply also implies that macroeconomic policies aimed at increasing the growth rate will have distributional implications. We have illustrated these by examining the distributional consequences of financing an investment subsidy. Our analysis suggests two important conclusions. First, it is possible to increase the growth rate and to reduce either gross or net income inequality. Second, it is often the case that fiscal policy has opposite effects on the distribution of gross and net income. These results highlight the fact that in addition to the usual tradeoff between equity and efficiency, policymakers concerned with the distribution of income may face a tradeoff between pre- and post-tax inequality. Understanding which types of inequalities agents and policy makers care about becomes essential, and raises the question of whether a slightly more unequal distribution of both gross and net incomes may, in certain cases, be a more desirable policy than a huge, but offset, increase in pre-tax inequality.

Finally, we conclude with a caveat. While the simple AK model has the advantage of providing a tractable framework for investigating the growth-inequality relationship and its policy implications, it also has the limitation that the economy is always on its balanced growth path. It therefore cannot address issues pertaining to the dynamics of wealth and income distribution. This is an important subject for further work. The analysis of this paper suggests that extending the present model to include human capital, thereby generating transitional dynamics as in Bond, Wang, and Yip (1996), is not only important in its on right, but may also provide a tractable framework for investigating this aspect.

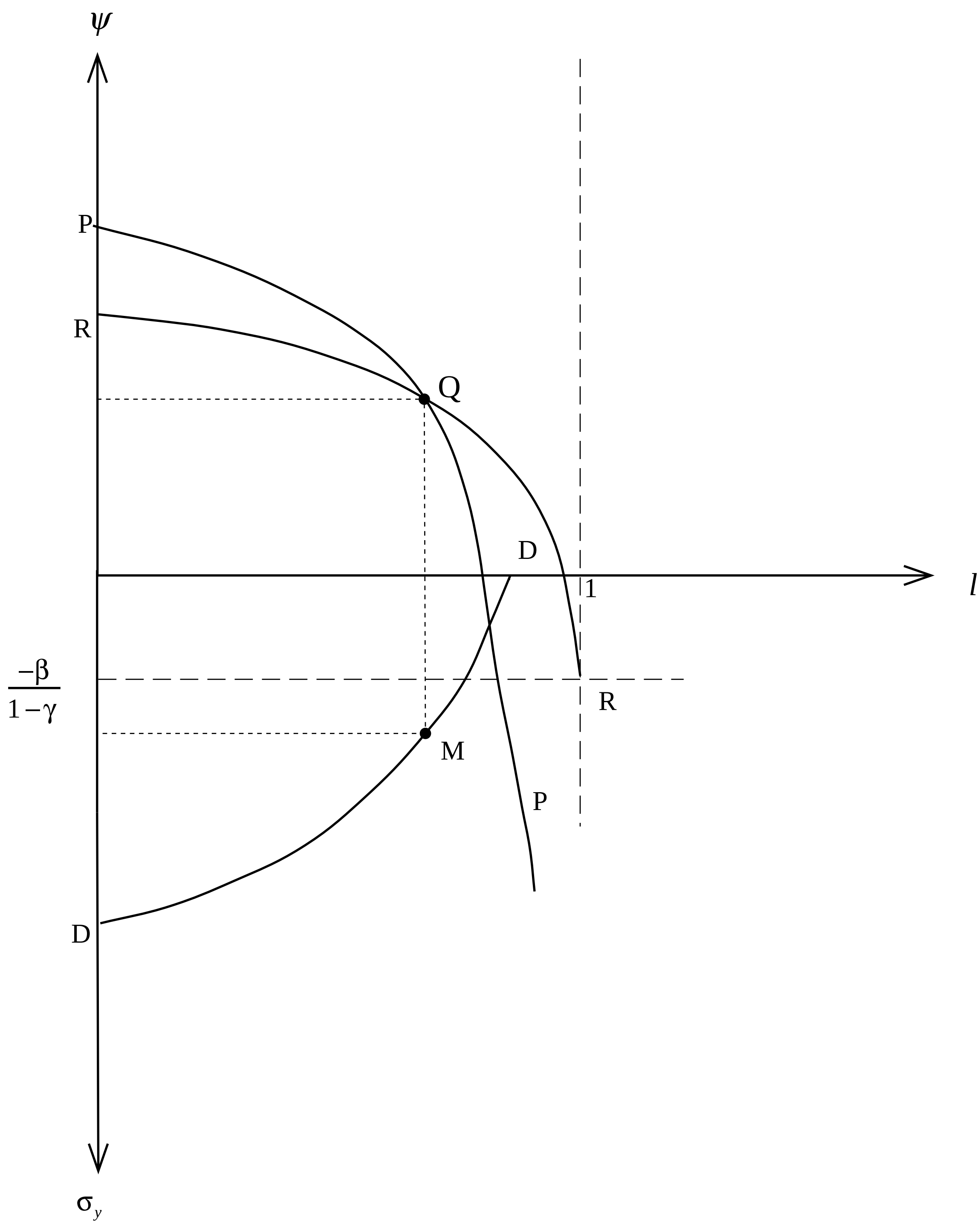

Fig 1: Equilibrium Growth, Employment, and Income Distribution

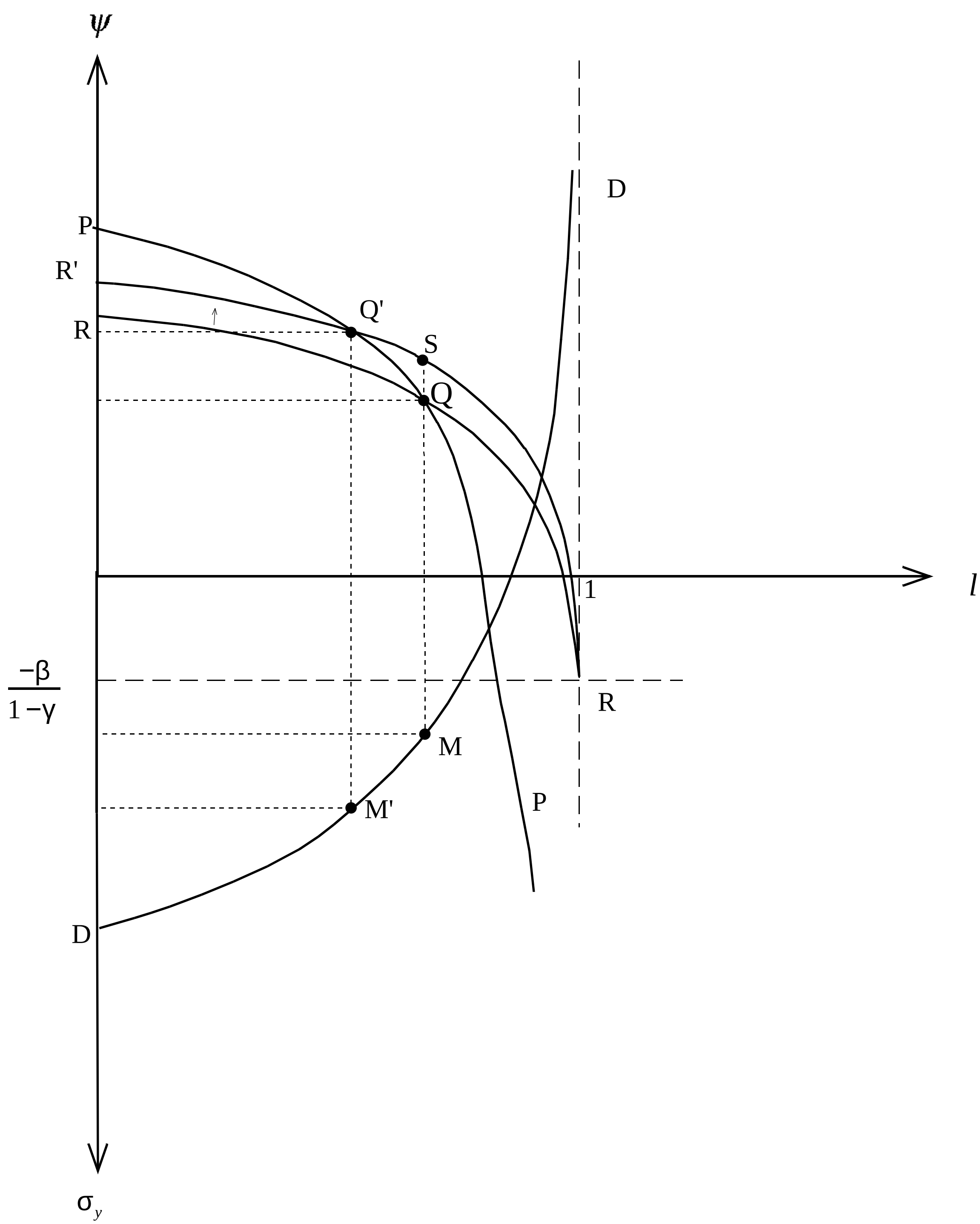

Fig 2: Decrease in Rate of Time Preference

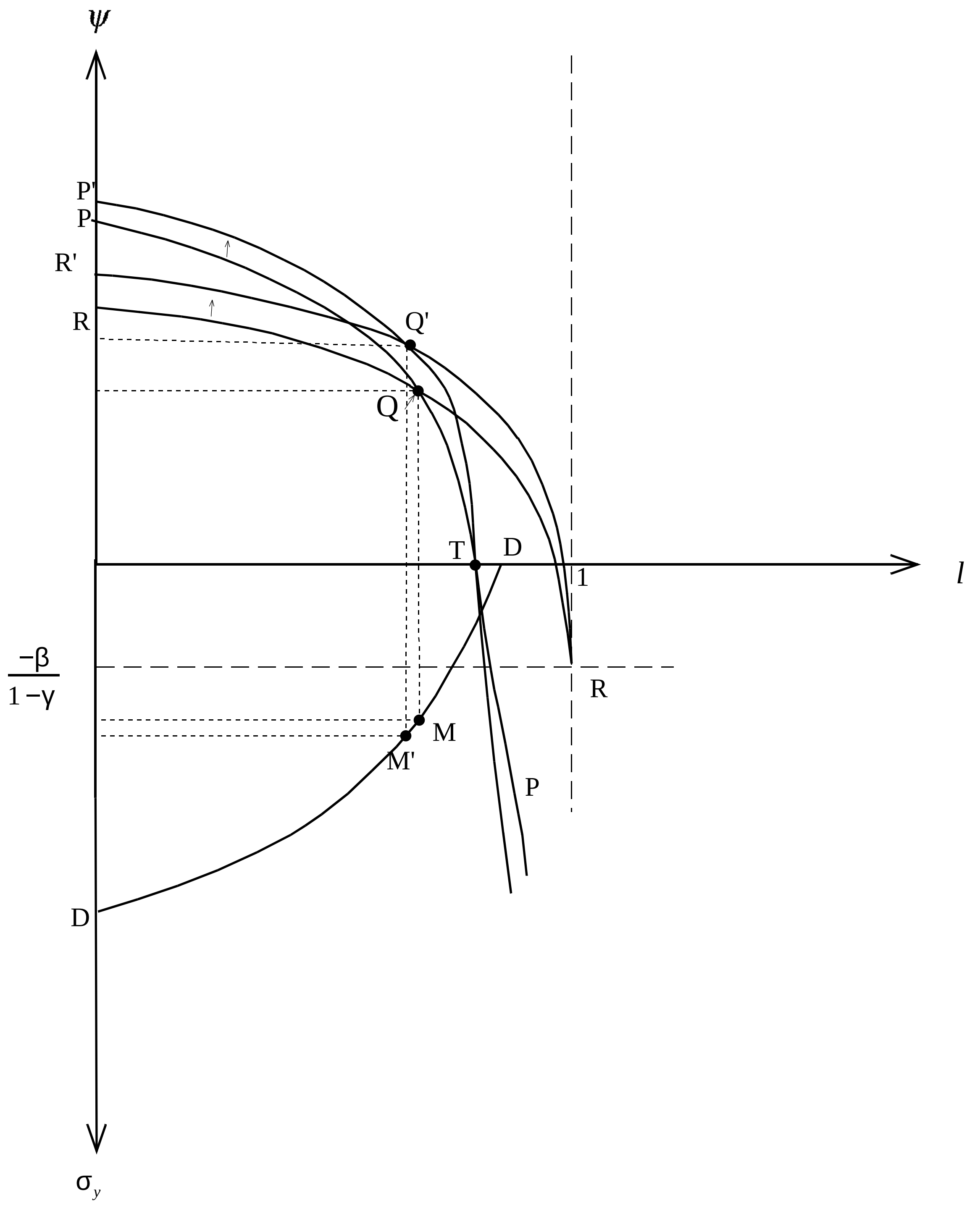

Fig 3: Increase in Productivity

Table 1

Effects of introducing investment subsidy under alternative modes of finance

	τ_k	τ_w	τ_c
ψ	$-\frac{\alpha^2\Omega^2}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]\left[1+\frac{1-\alpha l}{1-l}\right]>0$	$\frac{\alpha\Omega^2}{D(1-l)}\left(\frac{1-\alpha}{1-\gamma}\right)\left(1+\frac{1}{\eta}-\frac{\alpha l}{\eta(1-l)}\left[1-\frac{l}{\eta(1-l)}\right]\right)>0$	$\frac{\alpha\Omega^2}{D(1-l)}\left(\frac{1-\alpha}{1-\gamma}\right)\frac{1}{\eta}>0$
l	$\frac{\alpha\Omega}{D(1-\gamma)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]<0$	$\frac{\Omega}{D}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$	$\frac{\Omega}{D}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]=-\frac{\beta}{D}<0$
ρ	$-\frac{\alpha\Omega(1-\rho)}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]>0$	$-\frac{\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$	$-\frac{\alpha\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]<0$
ρ^N	$-\frac{\alpha\Omega(1-\rho)}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]$ $-(1-\rho)\left[1-\frac{\alpha}{\eta}\frac{l}{1-l}\right]$	$-\frac{\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$ $+\frac{(1-\alpha)(1-\rho)}{\alpha}\left[1-\frac{\alpha}{\eta}\frac{l}{1-l}\right]$	$-\frac{\alpha\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]<0$

$D\equiv\frac{\alpha\Omega}{(1-l)}\left(1+\frac{1-\alpha l}{\eta(1-l)}-\frac{1-\alpha}{1-\gamma}\right)$, assumed to be >0

Table 2

Relative Rankings of Alternative Modes of Financing Investment Subsidy

Growth $$\left.\frac{\partial \psi}{\partial s}\right|_{\tau_c} > \left.\frac{\partial \psi}{\partial s}\right|_{\tau_w} > \left.\frac{\partial \psi}{\partial s}\right|_{\tau_k} > 0$$

Leisure $$\left.\frac{\partial l}{\partial s}\right|_{\tau_k} < \left.\frac{\partial l}{\partial s}\right|_{\tau_c} < \min\left[\left.\frac{\partial l}{\partial s}\right|_{\tau_w}, 0\right];$$

sufficient condition for $\left.\frac{\partial l}{\partial s}\right|_{\tau_w} > 0$ is $\tilde{l} < \frac{\eta'}{\alpha + \eta'}$ where $\eta' \equiv \eta\left(\frac{\alpha - \gamma}{1 - \gamma}\right)$

Gross income inequality $$\left.\frac{\partial \rho}{\partial s}\right|_{\tau_k} > \left.\frac{\partial \rho}{\partial s}\right|_{\tau_c} > \max\left[\left.\frac{\partial \rho}{\partial s}\right|_{\tau_w}, 0\right]$$

sufficient condition for $\left.\frac{\partial \rho}{\partial s}\right|_{\tau_w} < 0$ is $\tilde{l} < \frac{\eta'}{\alpha + \eta'}$ where $\eta' \equiv \eta\left(\frac{\alpha - \gamma}{1 - \gamma}\right)$

Net income inequality $$\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_w} > \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_c} > \max\left[\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_k}, 0\right]$$

Sufficient condition for $\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_c} > \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_k}$ is $\eta < \frac{\alpha}{(1-\alpha)^2}$

Gross versus net income inequality $$\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_w} > \left.\frac{\partial \rho}{\partial s}\right|_{\tau_w}; \quad \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_c} = \left.\frac{\partial \rho}{\partial s}\right|_{\tau_c}; \quad \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_k} < \left.\frac{\partial \rho}{\partial s}\right|_{\tau_k}$$

Table 3
Growth, Distribution of Income and Welfare

	l	ψ	ρ	Gini(y)	Δ(X)	Δ(Y)
$A=0.75, \beta=0.04$	72.5	3.27	0.206	33.27	-149.2	-163.2
$A=1.0, \beta=0.04$	72.3	4.84	0.212	33.65	50.13	50.35
$A=0.75, \beta=0.02$	72.1	3.98	0.218	34.02	-4.99	-4.62
$A=0.75, \beta=0.04$ $\alpha=0.50$	75.3	4.88	0.265	36.86	26.37	23.05
$A=0.75, \beta=0.04$ $\eta=1.25$	65.2	3.97	0.233	34.94	25.36	25.26

Table 4
First-best Taxation

	s	τ_w $(=-\tau_c)$	τ_k	l	ψ	ρ	ρ^N	Gini(y)	Gini(ny)	Δ(X)	Δ(Y)
	0	-592.0	-150.0							(-102.7)	(-104.4)
	30.0	-384.4	-75.0								
$A=0.75$ $\beta=0.04$	60.0	-176.8	0	67.2	11.48	0.336	0.107	40.99	27.10	20.53	25.03
	85.42	0	63.55								
	90.0	30.22	75.0								
	0	-754.6	-150.0								
	30.0	-498.3.	-75.0								
$A=1.0$ $\beta=0.04$	60.0	-241.9	0	66.5	16.64	0.349	0.093	41.79	26.14	46.92	47.32
	88.15	0	70.39								
	90.0	14.50	75.0								
	0	-715.7	-150.0							(-102.7)	(-104.4)
	30.0	-471.0	-75.0								
$A=0.75$ $\beta=0.02$	60.0	-226.3	0	66.6	12.27	0.347	0.096	41.63	26.35	-3.68	-3.45
	87.50	0	68.75								
	90.0	18.40	75.0								

Table 5
Arbitrary Taxation

	τ_k	τ_w $(=-\tau_c)$	τ_c	l	ψ	ρ	ρ^N	Gini(y)	Gini(ny)	Δ(X)	Δ(Y)
$s=0$	0	0	0	72.5	3.27	0.206	0.206	33.27	33.27	--	--
$s=30$	10.01	0	0	71.6	4.70	0.230	0.198	34.77	32.81	7.63	8.43
$s=30$	0	7.58	0	72.8	5.21	0.198	0.223	32.77	34.33	9.60	9.34
$s=30$	0	0	5.38	72.3	5.28	0.213	0.213	33.72	33.72	10.00	10.24
$s=30$	0	-18.58	18.58	71.2	5.44	0.243	0.192	35.52	32.43	10.68	11.91

Table 6
Second Best

	s	τ_k	τ_w $(=-\tau_c)$	τ_c	l	ψ	ρ	ρ^N	Gini(y)	Gini(ny)	Δ(X)	Δ(Y)
base	0	0	0	0	72.5	3.27	0.206	0.206	33.27	33.27	--	--
Max ΔX	85.6	64.0	0	0	67.2	11.49	0.336	0.107	40.99	27.10	20.53	25.03
Max ΔY	88.6	70.0	0	0	66.6	12.28	0.347	0.092	41.63	26.12	20.40	25.18
Max ΔX	57.3	0	26.00	0	74.2	8.33	0.156	0.260	30.18	36.55	16.72	14.95
Max ΔY	54.7	0	23.21	0	73.9	9.09	0.164	0.254	30.69	36.21	16.59	15.10
Max ΔX	60.2	0	0	25.41	72.0	10.37	0.221	0.221	34.18	34.18	18.81	19.35
Max ΔY	60.4	0	0	25.41	72.0	10.43	0.221	0.221	34.18	34.18	18.81	19.35

Appendix

This appendix provides some of the conditions required to ensure the existence of a balanced growth equilibrium underlying the derivations of the equilibrium conditions.

A.1 Decentralized economy

It suffices to focus on the economy without taxation; the introduction of taxes leads to minor modifications and can be analyzed analogously. Differentiating the relations in (19a') and (19b'), we obtain

$$\left.\frac{\partial \psi}{\partial l}\right|_{RR} = -\frac{\alpha(1-\alpha)\Omega(l)}{1-l}\frac{1}{1-\gamma} < 0, \qquad \left.\frac{\partial^2 \psi}{\partial l^2}\right|_{RR} = -\frac{\alpha(1-\alpha)^2\Omega(l)}{(1-l)}\frac{1}{1-\gamma} < 0 \qquad \text{(A.1a)}$$

$$,\left.\frac{\partial \psi}{\partial l}\right|_{PP} = -\frac{\alpha\Omega(l)}{(1-l)}\left(1+\frac{1-\alpha l}{\eta(1-l)}\right) < 0 \qquad \left.\frac{\partial^2 \psi}{\partial l^2}\right|_{PP} = -\frac{\alpha(1-\alpha)\Omega(l)}{(1-l)^2}\left(1+\frac{2-\alpha l}{\eta(1-l)}\right) < 0 \qquad \text{(A. 1b)}$$

so that both the (PP) and (RR) have negative slopes and are strictly concave. Also

$$\psi_{RR}(l=0) = \frac{A(1-\alpha)-\beta}{1-\gamma}, \qquad \psi_{RR}(l=1) = -\frac{\beta}{1-\gamma}$$
$$\psi_{PP}(l=0) = A, \qquad \psi_{PP}(l=1) = -\infty.$$

Given concavity, sufficient condition to ensure the existence of a unique equilibrium are: (i) $\psi_{PP}(l=0) > \psi_{RR}(l=0)$, so that the (PP) schedule lies above the (RR) curve at $l=0$, and (ii) the (PP) schedule is steeper at each point than is the RR schedule.. In this case, the (PP) schedule lies below (RR) for $l=1$, and the two schedules only cross once. Condition is satisfied when

$$\alpha - \gamma + \frac{\beta}{A} > 0, \qquad \text{(A.2a)}$$

while condition (ii) requires

$$1+\frac{1-\alpha l}{\eta(1-l)} > \frac{1-\alpha}{1-\gamma}. \qquad \text{(A.2b)}$$

Both of these are very weak conditions and will certainly hold if $\gamma < 0$, as we are assuming.

A.2 The centrally planned economy

The two equilibrium conditions are (19a") and (19b'). In this case, we obtain

$$\left.\frac{\partial \psi}{\partial l}\right|_{R'R'} = -\frac{\Omega(l)}{1-l}\left[\frac{1}{1-\gamma}\right] < 0\,, \quad \left.\frac{\partial^2 \psi}{\partial l^2}\right|_{RR"} = -\frac{\alpha\Omega(l)}{(1-l)^2}\left[\frac{1-\alpha}{1-\gamma}\right]. \qquad \text{(A.3)}$$

so that the (R'R') is also concave. The sufficient conditions for the existence of a unique equilibrium, (i) and (ii) are now modified to:

$$-\gamma + \frac{\beta}{A} > 0\,. \qquad \text{(A.2a'))}$$

$$1 + \frac{1-\alpha l}{\eta(1-l)} > \frac{1}{1-\gamma}\,. \qquad \text{(A.2b')}$$

both of which still are met if $\gamma < 0$.

References

Aghion, P, E. Caroli, and C. García-Peñalosa, 1999, "Inequality and Economic Growth: The Perspective of the New Growth Theories," *Journal of Economic Literature* 37, 1615-1660.

Aghion, P. and P. Bolton. 1997. "A Trickle-Down Theory of Growth and Development with Debt Overhang," *Review of Economic Studies*, 64, 151-72.

Alesina and Rodrik, 1994, "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 109, 465-490.

Anand, S. and R. Kanbur, 1993, "The Kuznets Process and the Inequality-Development Relationship," *Journal of Development Economics*, 40, 25-52.

Atkinson, A.B., 2003, "Income Inequality in OECD Countries: Data and Explanations," *CESifo Economic Studies*, 49, 479-513.

Barro, R.J., 2000, "Inequality and Growth in a Panel of Countries," *Journal of Economic Growth*, 5, 5-32.

Benabou, R., 1996, "Equity and Efficiency in Human Capital Investment: The Local Connection," *Review of Economic Studies*, 63, 237-264.

Bertola, G., 1993, "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 83, 1184-98.

Bond, E.W., P. Wang, and C.K. Yip, 1996, "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics," *Journal of Economic Theory*, 68, 149-173.

Bourguignon, F., 1981, "Pareto-Superiority of Unegalitarian Equilibria in Stiglitz's Model of Wealth Distribution with Convex Savings Function," *Econometrica*, 49, 1469-1475.

Breen, R. and C. García-Peñalosa, 2004, "Income Inequality and Macroeconomic

Caselli, F., and J. Ventura, 2000, "A Representative Consumer Theory of Distribution," *American Economic Review*, 90, 909-926.

Chamley, C., 1986, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica*, 54: 607-622.

Checchi, D. and C. Garcia Penalosa, 2004, "Risk and the Distribution of Human Capital," Economics Letters, 82, 53-61.

Correia, I.H., 1999, "On the Efficiency and Equity Trade-off," *Journal of Monetary Economics*, 44, 581-603.

Deininger, K. and Squire, L., 1996. "Measuring Income Inequality: a New Data Base," *The World Bank Economic Review* 10, 565-591.

Forbes, K., 2000, "A Reassessment of the Relationship between Inequality and Growth," *American Economic Review*, 90, 869-887.

Fuest, C., and B. Huber, 2001, "Labor and Capital Income Taxation, Fiscal Competition, and the Distribution of Wealth," *Journal of Public Economics*, 89: 71-91.

Galor, O. and J. Zeira, 1993, "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60, 35-52.

Galor, O. and Tsiddon, 1997, "Technological Progress, Mobility, and Economic Growth," *American Economic Reivew*, 87, 363-382.

García-Peñalosa, C, and S.J. Turnovsky, 2004, "Second-Best Optimal Taxation of Capital and Labor in a Developing Economy," *Journal of Public Economics*, forthcoming.

Gylfason, T. and G. Zoega, 2003, "Inequality and Growth: Do Natural Resources Matter?" in *Inequality and Growth: Theory and Policy Implications*: T.S. Eicher and S.J. Turnovsky (eds.), Cambridge MA: MIT Press.

Judd, K.L, 1985, "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28: 59-83.

Kormendi, R. and P. Meguire, 1985, "Macroeconomic Determinants of Growth: Cross-Country Evidence," *Journal of Monetary Economics*, 16, 141-63.

Koskela, E and R. Schöb 2002. "Optimal factor Income Taxation in the Presence of Unemployment," *Journal of Public Economic Theory*, 4: 387-404.

Kuznets, S., 1955, "Economic Growth and Income Inequality," *American Economic Review*, 45, 1-28.

Li, H.Y. and H.F. Zou, 1998, “Income Inequality is not Harmful to Growth: Theory and Evidence” *Review of Development Economics*, 2, 318-334.

Lundberg, M. and L. Squire (2003), “The Simultaneous Evolution of Growth and Inequality,” *Economic Journal*, 113, 2003, 326-344.

MaCurdy, T., 1981, “An Empirical Model of Labor Supply in a Life Cycle Setting,” *Journal of* MaCurdy, T., 1981, “An Empirical Model of Labor Supply in a Life Cycle Setting,” *Journal of Political Economy*, 89, 1059-1085.

Perotti, R., 1996, “Growth, Income Distribution, and Democracy: What the Data Say,” *Journal of Economic Growth*, 1, 149-187.

Persson, T. and G. Tabellini, 1994, “Is Inequality Harmful for Growth?” American Economic Review, 84, 600-621.

Romer, P.M., 1986, “Increasing Returns and Long-Run Growth,” *Journal of Political Economy*, 94, 1002-37.

Stiglitz, J., 1969, “The Distribution of Income and Wealth Among Individuals,” *Econometrica*, 37, 382-397.

Turnovsky, S.J., 2000, “Fiscal Policy, Elastic Labor Supply, and Endogenous Growth,” *Journal of Monetary Economics*, 45, 185-210.

Viane, J.-M. and I. Zitcha, 2003, “Human Capital formation, Income Inequality, and Growth,” in *Inequality and Growth: Theory and Policy Implications*: T.S. Eicher and S.J. Turnovsky (eds.), Cambridge MA: MIT Press.

Wolff, E.N., 1998, “Recent Trends in the Size Distribution of Household Wealth,” *Journal of Economic Perpectives*, 12, 131-50.

Wood, A., 1994, North-South Trade, Employment and Inequality: Changing Fortunes in a Skill Driven World, Oxford: Clarendon Press.